One-dimensional inverse problem for wave equation

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We consider wave equation with inhomogeneity $\sigma(x)u_t^m + q(x)u^p$, $m > 1$, $p > 1$, and study a forward and an one-dimensional inverse problems for this equation.

Let T be a real positive number.

A forward problem. Determine the function $u(x,t)$ satisfying the relations

$$u_{tt} - u_{xx} - \sigma(x)u_t^m - q(x)u^p = 0, \quad x > 0, \quad t \in (0,T]; \tag{1}$$

$$u|_{t=0} = u_t|_{t=0} = 0, \tag{2}$$

$$u|_{x=0} = f(t), \tag{3}$$

where $\sigma(x)$ and $q(x)$ are continuous functions; $m > 1$ and $p > 1$ are real numbers; $f(t)$ is the twice continuously differentiable function and $f(0) = a > 0$, $f'(0) = b > 0$, a, b are some constants.

An inverse problem. Let $f_k(t)$, $k = 1, 2$, be the given functions and $f_k(0) = a_k > 0$, $f_k'(0) = b_k$, $k = 1, 2$, numbers a_k and b_k satisfy the condition $a_1^p b_2^m - a_2^p b_1^m \neq 0$. The solution of the forward problem (1)–(3) for $f = f_k$, $k = 1, 2$ denote $u^k(x,t)$, $k = 1, 2$. Find the functions $\sigma(x)$ and $q(x)$ from the given information about solutions $u^k(x,t)$:

$$u_x^k|_{x=0} = h_k(t), \quad t \in [0,T], \quad k = 1, 2. \tag{4}$$

Conditions for the unique solvability of the forward problem are found. For the inverse problem a local existence and uniqueness theorems are established. Both theorems for forward and inverse problems are new in the theory of inverse problems (see paper [1]).

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References

1. *Romanov V.G., Bugueva T.V.* An one-dimensional inverse problem for the wave equation // Euras. J. Math. Comput. Appl. 2024. V. 12. No. 3. P. 135–162.