Modelling slurry flow in a hydraulic fracture: from mathematical model to deployable software

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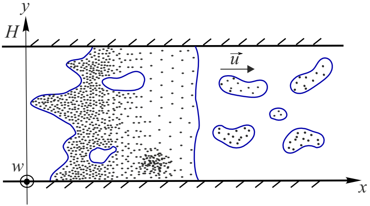
Mathematical model



Governing equations describing the flow

$$\frac{\partial w}{\partial t} + \nabla \cdot (w\mathbf{u}) = -q_l \left(1 - \frac{c_p}{c_{max}}\right); \quad \frac{\partial (c_p w)}{\partial t} + \nabla \cdot (c_p w \mathbf{u}_p) = 0.$$

The effective viscosy approach with single velocity will be used here:

$$\mathbf{u} = \mathbf{u}_f = \mathbf{u}_p = -\Lambda(|\nabla p|, c_p, w)(\nabla p + \rho g e_y); \quad \Lambda = \frac{w^2}{12\mu(c_p)}. \qquad (1)$$

Discretization

For the sake of brevity, we don't consider proppant settling velocity and the gravity terms are also omited. This leads to the following equations:

$$\frac{\partial}{\partial x}\Big(\Lambda \frac{\partial p}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(\Lambda \frac{\partial p}{\partial y}\Big) = \bar{f}, \tag{2}$$

$$\frac{\partial}{\partial t}(wc_p) + \nabla \cdot (wc_p \mathbf{u}) = 0. \tag{3}$$

Introducing the computational grid, (2) is integrated over each cell (FVM):

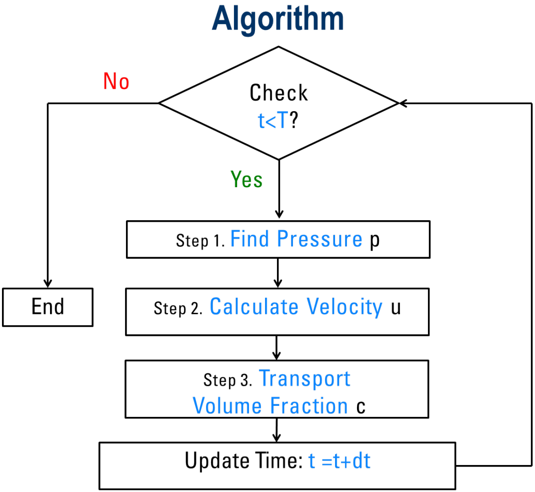
$$p_{i-1,j}\Lambda_{i-\frac{1}{2},j}\frac{h_y}{h_x} + p_{i,j-1}\Lambda_{i,j-\frac{1}{2}}\frac{h_x}{h_y} + p_{i+1,j}\Lambda_{i+\frac{1}{2},j}\frac{h_y}{h_x} + p_{i,j+1}\Lambda_{i,j+\frac{1}{2}}\frac{h_x}{h_y} -$$
$$-p_{i,j}\left[(\Lambda_{i-\frac{1}{2},j} + \Lambda_{i+\frac{1}{2},j})\frac{h_y}{h_x} + (\Lambda_{i,j-\frac{1}{2}} + \Lambda_{i,j+\frac{1}{2}})\frac{h_x}{h_y}\right] = \bar{f}_{i,j} h_x h_y.$$

Proppant transport (3) is modelled using numerical scheme (by R.J. Leveque):

$$(cw)^{n+1}_{i,j} = (cw)^n_{i,j} - \frac{\Delta t}{h_x}(F_{i+\frac{1}{2},j} - F_{i-\frac{1}{2},j}) - \frac{\Delta t}{h_y}(G_{i,j+\frac{1}{2}} - G_{i,j-\frac{1}{2}}).$$

These equations are tied by a certain formula for effective viscosity $\mu = \mu(c)$.

Application and extensions



Application and extensions

Features and bottlenecks

- The implemented algorithm provides flux coherence (stencil of the FVM adjusts on border nodes)
- To find the pressure distribution SLAE must be solved
- The SLAE matrix is a sparse 5-diagonal matrix, but very poorly conditioned
- Big sensitivity to volume balance violation (on poisson equation part)
- No ghost cells are needed, but boundary cells must be dealt with caution
- The problem of implementing fracture geometry

For this very simple model numerous extensions can be applied:

- Non–Newtonian fluids
- Proppant settling
- Transition to a multiphase model (fluids and proppants)
- Proppant bridging criterion
- Thermodynamical effects
- etc.

Implementation

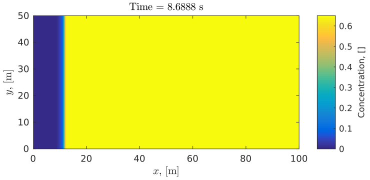
The model has been coded using C++, cmake, python, Gtest, git. By choosing this stack we benefit from power and flexibility in exchange for complexity

- Cross-platform development and deployment
- Ability to use external libraries for acceleration
- Automated testing
- Pack into .dll or use the standalone version

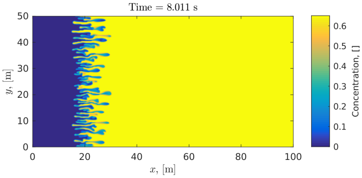
Tips painfully derived from our team's experience

- Always check the math (we are still doing science)
- Keep in mind technical complexity (programming is a craft)
- Create architecture before-head, use best programming practices
- Use modern tools and compilers, don't neglect tests, try python
- Create the most simple and versatile (but ugly) interface to the module
- Avoid writing your own standard or common methods and algorithms
- Use git

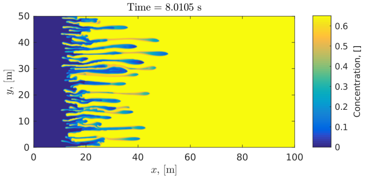
Viscosity relation μ_2/μ_1 effects the instability. $t \approx 8$s



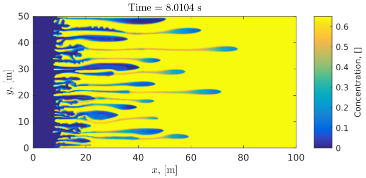
(а) $\mu_2/\mu_1 = 0.1$



(б) $\mu_2/\mu_1 = 10$

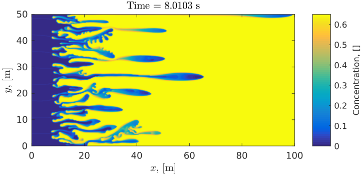


(в) $\mu_2/\mu_1 = 31$

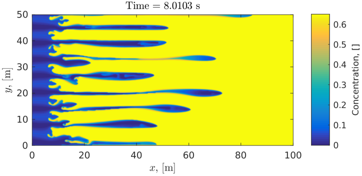


(г) $\mu_2/\mu_1 = 100$

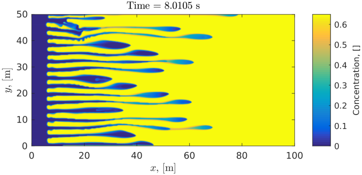
Proppant concentration perturbation $c_p = \varepsilon c_{max} \left(\sin\left(\frac{2\pi \mathbf{m} y}{L_y} \right) + 1 \right)$ affecting the formation of viscous fingering. $\mu_2/\mu_1 = 31$, $t \approx 8$c.



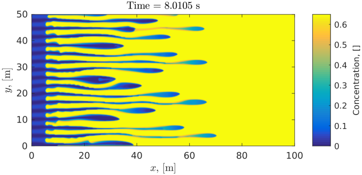
(a) $m = 8; \varepsilon = 0.01$



(b) $m = 8; \varepsilon = 0.05$

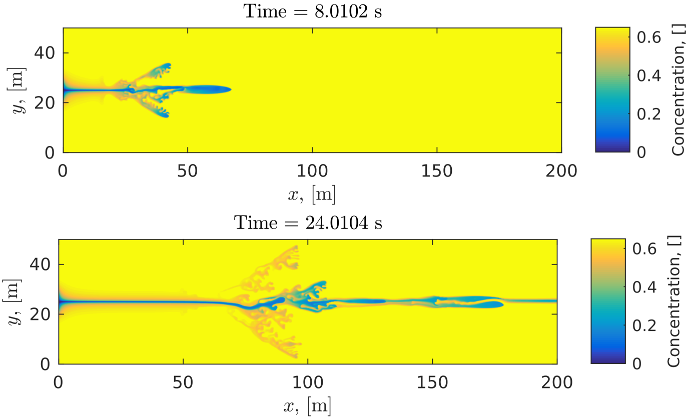


(c) $m = 16; \varepsilon = 0.01$



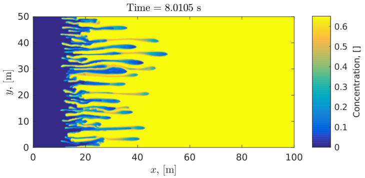
(d) $m = 16; \varepsilon = 0.05$

Single finger propagation

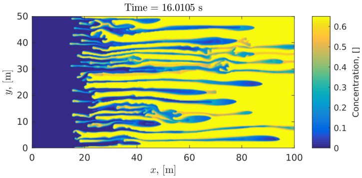


Rough walls simulation

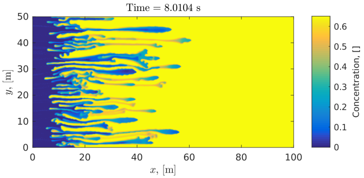
Random walls perturbations with amplitude ε^*.



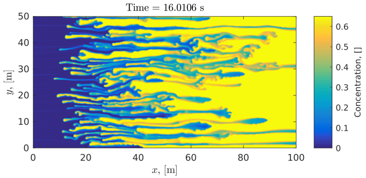
(a) $\varepsilon^* = 0$; $t \approx 8$ c



(b) $\varepsilon^* = 0$; $t \approx 16$ c

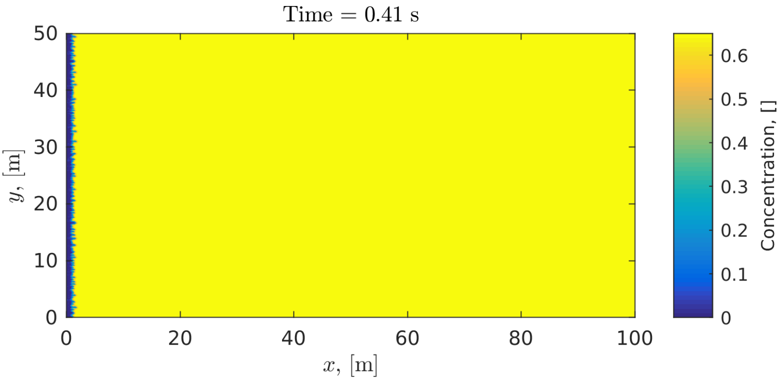


(c) $\varepsilon^* = 0.05$; $t \approx 8$ c



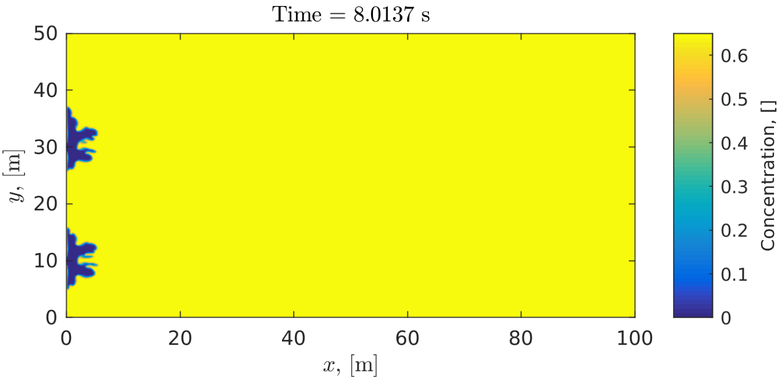
(d) $\varepsilon^* = 0.05$; $t \approx 16$ c

Rough walls simulation



perturbation amplitude $\varepsilon^* = 0.05$, viscosity relation$\mu_2/\mu_1 = 31$

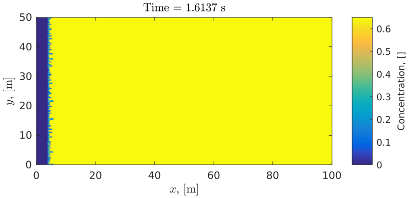
Fluid inflow through a perforation



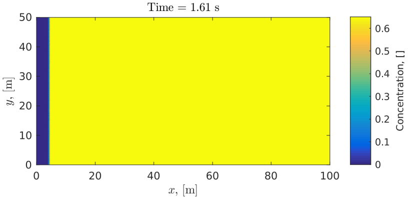
Viscosity relation is $\mu_2/\mu_1 = 31$

Non-Newtonian fluid

Fluid with power-law rheology, consistency indexes contrast is $K_2/K_1 = 100$

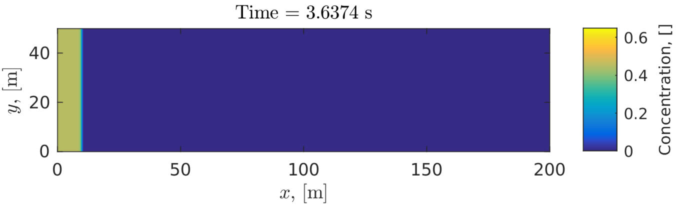


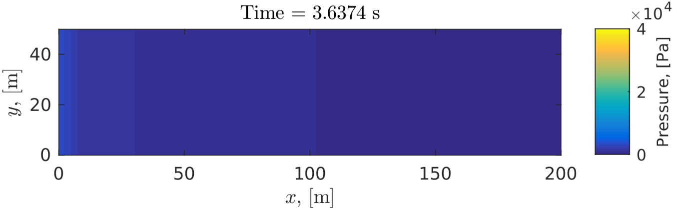
(a) n = 0.5



(b) n = 0.7

Non zero Leak-off

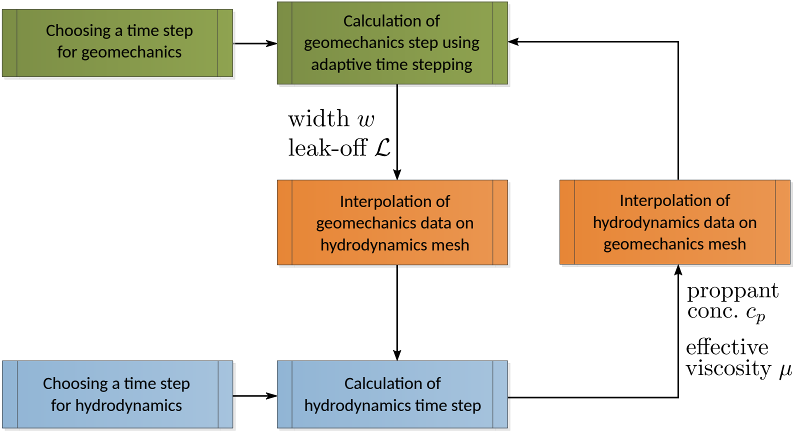




Coupling with the geomechanical model

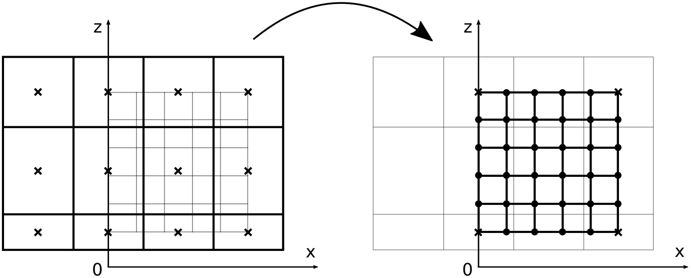
We try to keep the hydrodynamics module architecturally and logically decoupled from other modules, the coupling has been performed with:

- Planar 3D ILSA model (report by Valov A.)
- EP3D model (report by Skopintsev A.)



Coupling with the geomechanics: problematics

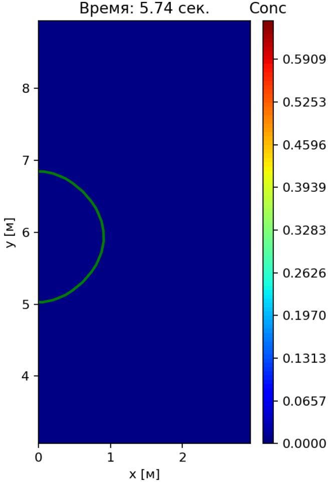
Interpolating data is essential. Bilinear algorithm can be considered, but a more advanced conservative method may become crucial.



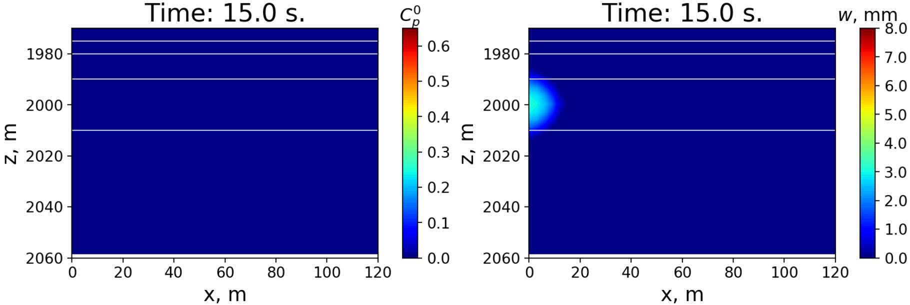
- Volume balance might be violated during interpolation
- Bringing fracture geometry onto hydrodynamics mesh
- Calculating on various time scales
- Duplicated calculations, starting with Poisson equation
- General lack of data in each module

Radial fracture with leak-off

Volumetric proppant concentration on the inlet is $c_p = 0.55$.



Pulsative injection



$E = 22$ GPa, $\nu = 0.23$, $\mu = 0.05$ Pa·s, $K_{I_C} = 1$ MPa·m$^{1/2}$,
$C_l = 2.0 \cdot 10^{-5}$ m·s$^{-1/2}$, $Q = 0.06$ m^3/s

Thank you for your attention!