Comparison of design optimization algorithms of multiply fractured well

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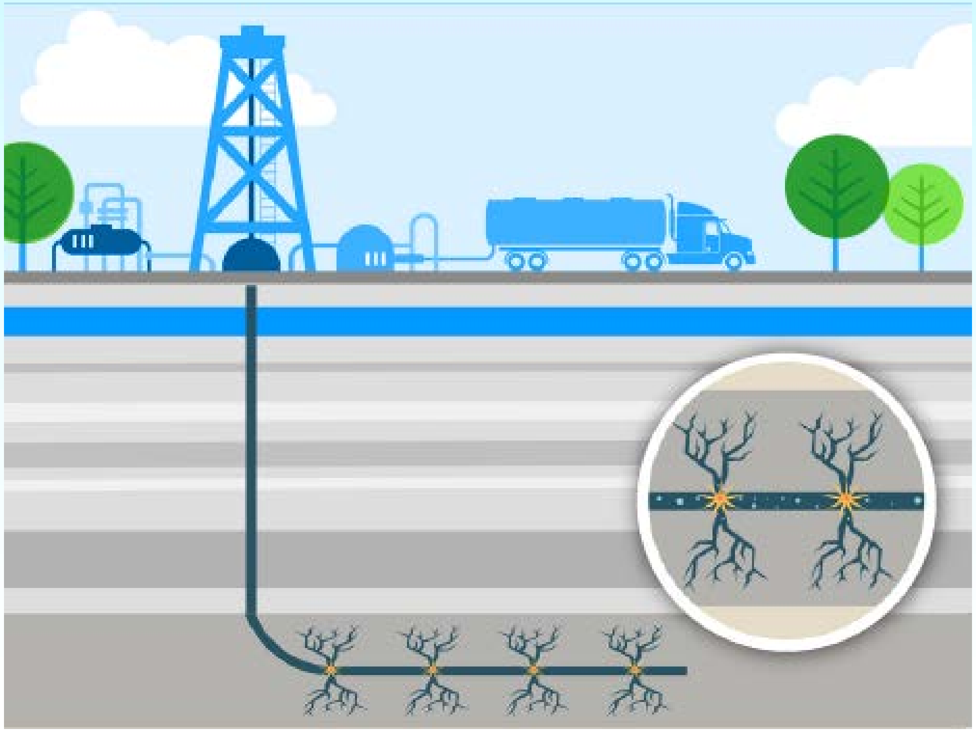
Hydraulic fracturing

Multiple fractured horizontal wells (MFHW) are widely used for enhancement of oil recovery of low permeability reservoirs.

MFHW design is characterized by a length and a width of fractures, a number of fractures and a length of a horizontal well.



The optimization problem

The problem of MFHW optimization can be formulated as following:

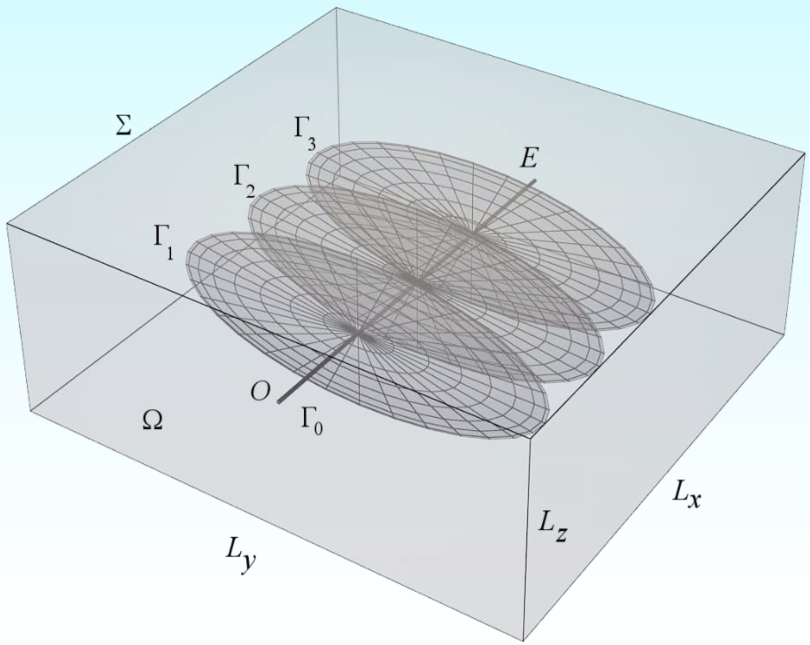
$$\begin{cases} \boldsymbol{f}(\boldsymbol{x}) = (C_{HF}, -NPV, -Q_{tot}) \rightarrow \text{min}, \\ \boldsymbol{x} = (N_f, M_p, L_w) \\ 4 \le N_f \le 12, \\ 4000 \le M_p \le 90000\ [kg], \\ 400 \le L_w \le 1200\ [m]. \end{cases}$$

Optimization parameters:

- the number of fractures N_f,
- the proppant mass for a fracture M_p,
- the length of a horizontal well L_w.

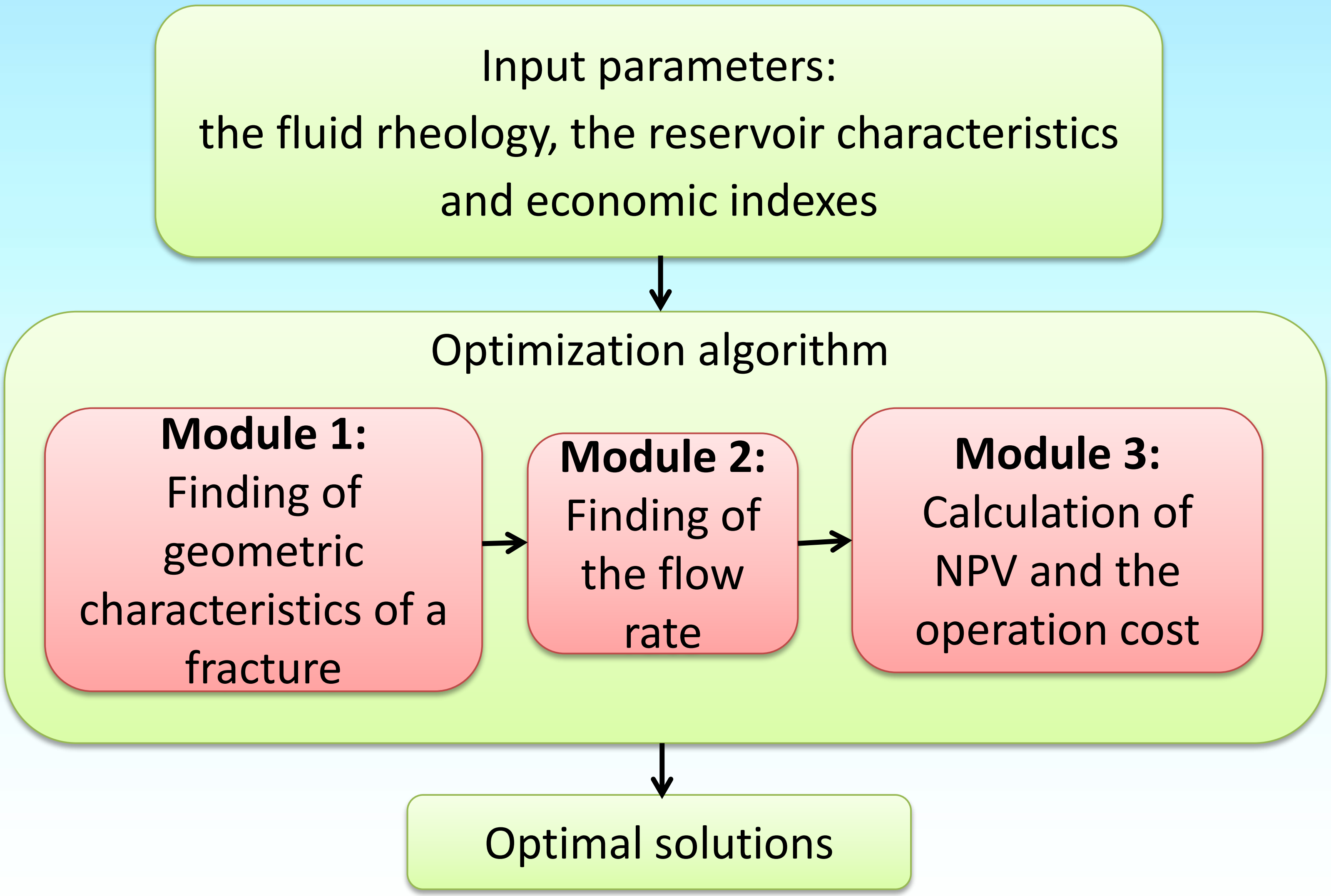
Optimization objectives:

- treatment costs C_{HF},
- Net Present Value NPV,
- cumulative well production Q_{tot}.



Scheme of MFHW

Scheme of solving the optimization problem



Module 1: Fracture geometry [1]

Volume balance for an incompressible Newtonian fluid inside the crack:

$$\frac{\partial w}{\partial t} + \frac{1}{r}\frac{\partial (rq)}{\partial r} + \frac{C'}{\sqrt{t - t_0(r)}} = Q_0\delta(r), \qquad q = -\frac{w^3}{\mu'}\frac{\partial p}{\partial r}$$

The elasticity equation:

$$p(r,t) = -\frac{E'}{2\pi R}\int_0^R M\left(\frac{r}{l},\frac{r'}{l}\right)\frac{\partial w(r',t)}{\partial r'}dr',$$

where the kernel is $M(\xi, s) = \begin{cases} \frac{1}{\xi}K\left(\frac{s^2}{\xi^2}\right) + \frac{\xi}{s^2-\xi^2}E\left(\frac{s^2}{\xi^2}\right), & \xi > s, \\ \frac{s}{s^2-\xi^2}E\left(\frac{\xi^2}{s^2}\right), & \xi < s. \end{cases}$

The functions *K* and *E* are the complete elliptic integrals of the first and the second kind, respectively.

The fracture width in the tip region:

$$w \to \frac{K'}{E'}(R-r)^{1/2}, \qquad r \to R$$

[1] E. V. Dontsov, «An approximate solution for a penny-shaped hydraulic fracture that accounts for fracture toughness, fluid viscosity and leak-off», Royal Society Open Science, 3: 160737, Published 7 December 2016.

Module 1: Fracture geometry

Input parameters:

T is the injection time,
μ_0 is the viscosity of injection fluid,
Q_0 is the injection rate,
E is the Young's modulus,
K_{Ic} is the mode I fracture toughness of the rock,
C_l is the Carter's leak-off parameter.

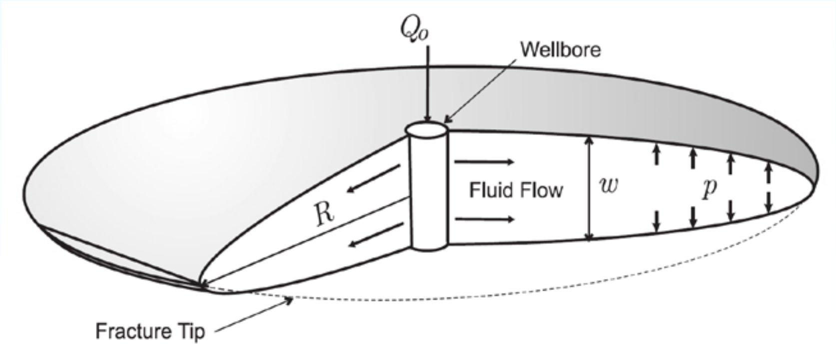
Output parameters:

l is the crack half-length,
w is the crack width.

$$T = \frac{M_p}{\rho_p Q_0 C}$$ is the injection time,

$$\mu = \mu_0 \left(1 - \frac{C}{C_{max}}\right)^{-2.5}$$ is the viscosity of proppant slurry.

Here ρ_p is the proppant density, μ_0 is the viscosity of carrier fluid, C is the proppant concentration.



Penny-shaped hydraulic fracture

Module 2: Calculation of the post-fracture production rate

Input parameters:

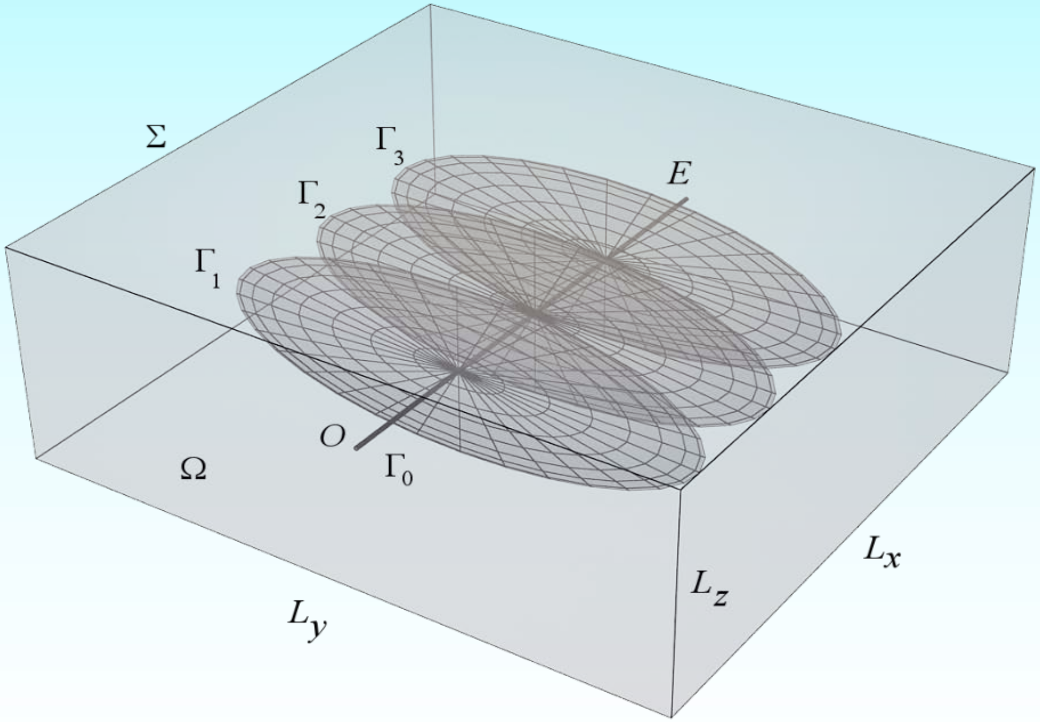
l is the crack half-length,

w *is* the crack width,

N_f is the number of fractures.

Output parameters :

Q_t is the production rate in time t.



Scheme of multiple fractured horizontal well

The express assessment of the production rate [2]

$$Q = \frac{2kHL}{b\mu_n(R-l)}\left(p_\text{п} - \left(\frac{1+2a}{1+a}\right)\frac{p_0}{2} - \left(\frac{1}{1+a}\right)\frac{p_\text{з}}{2}\right) + \frac{2\pi kH(p_\text{п} - p_\text{з})}{b\mu_n \ln\left(\frac{2R}{r_s}\right)}$$

$F_{cd} = \dfrac{k_f w}{kl}$ is the dimensionless fracture conductivity,

$p_0 = \dfrac{p_\text{п}(1-a) - \left(\frac{1}{2} - \bar{b}\right)p_\text{з}}{\frac{1}{2} + \bar{b} + a}$ is the intermediate pressure,

$$a = \frac{2l\left(N_f - 1\right)}{LF_{cd}}, \qquad \bar{b} = \frac{4\left(N_f - 1\right)^2 l(R-l)}{L^2}.$$

We assume that the post-fracture production rate declines exponentially

$$Q_t = Qe^{-\alpha t}.$$

[2] S. V. Elkin, A. A. Aleroev, N. A. Veremko and M. V. Chertenkov, «Model for the rapid calculation of the flow rate of the horizontal well fluid as a function of the number of hydraulic fracturing cracks», Oil Industry Journal, №12, 2016.

Module 3: Economic criteria

The economic criterion NPV is calculated as following:

$$NPV = \sum_{t=1}^{T_{max}} \frac{\Pi_t - A_t}{(1+D)^t} - C_{HF} \, .$$

Here Π_t is the cash inflow at t-th year, A_t is current expenses, D is the discount rate, T_{max} is the number of years which a revenue is calculated for.

We propose to estimate the fracturing cost as following:

$$C_{HF} = N_f \Big(\underbrace{Pr_p \cdot M_p}_{\text{proppant cost}} + \overbrace{TC \cdot M_p}^{\text{proppant injection costs}} + \underbrace{V_F \cdot Pr_F}_{\text{fluid cost}} \Big) + \underbrace{DCost \cdot L_w}_{\text{drilling costs}} + \overbrace{AC}^{\text{additional costs}} .$$

Here N_f is the number of fractures, Pr_p is the proppant price, M_p is the proppant mass, TC is the cost of a proppant injection, V_F is the volume of the fluid, Pr_F is the fluid price, $DCost$ is the drilling cost, L_w is the length of a horizontal well and AC is the fixed and miscellaneous costs.

Optimization algorithms

In our case a potential solution is $x = (N_f, M_p, L_w)$.

1. The number of criteria $M = 1$:
   - If $f(x_1) < f(x_2)$: x_1 is better than x_2.

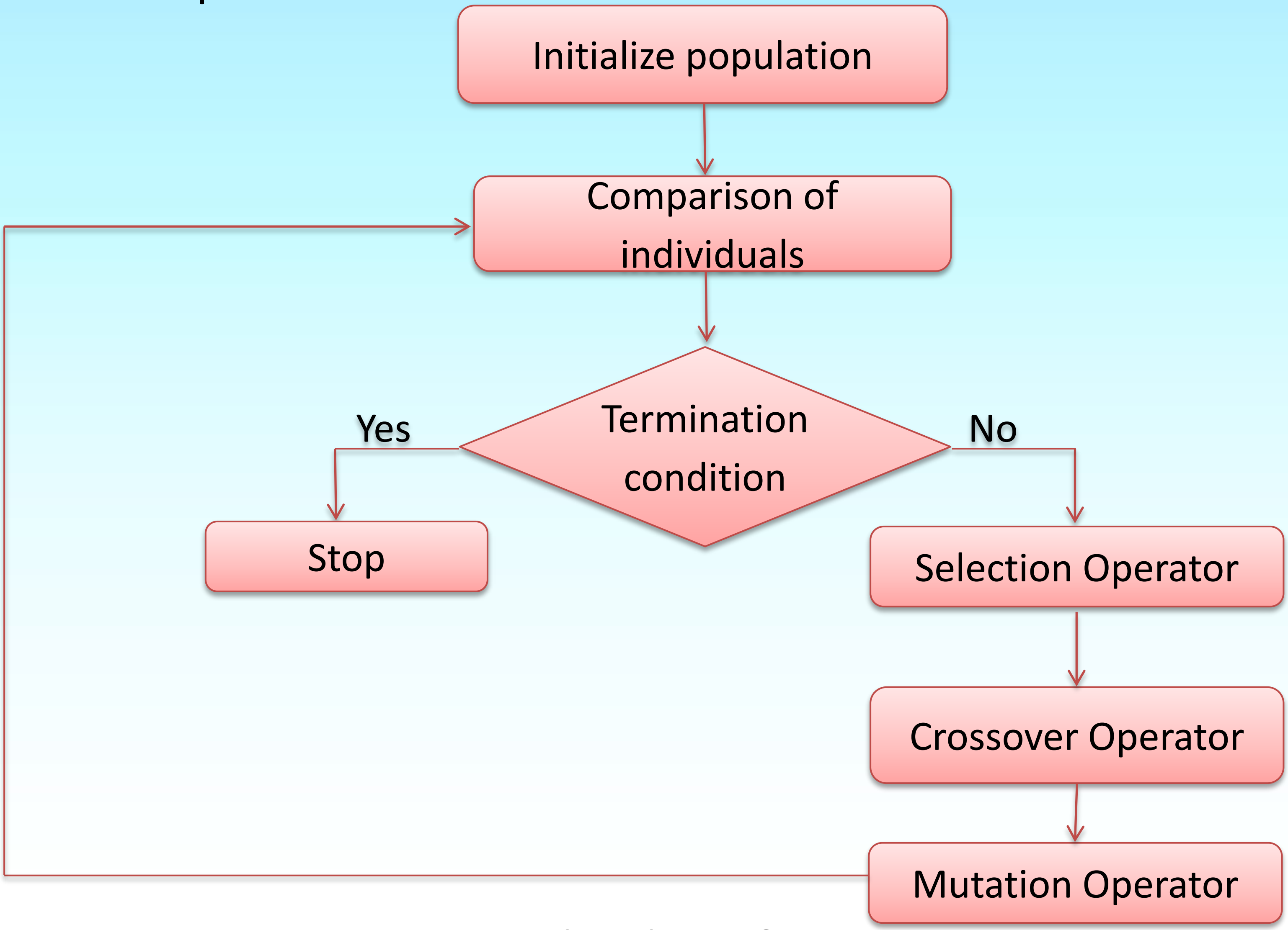
2. $\mathrm{M} > 1$:
   - x_1 **dominates** x_2, if $f_i(x_1) < f_i(x_2)\ \forall i$.
   - If x_1 is better than x_2 for one criterion, but x_2 is better for another one: x_1 and x_2 are **non-dominant**.
   - The set of non-dominate solutions is called **Pareto front**.

Applied three different stochastic algorithms:

- **the genetic algorithm NSGA-II,**
- **the particle swarm optimization,**
- **the simulated annealing.**

Genetic algorithm

A genetic algorithm (GA) is an evolutionary search algorithm that emulates the process of natural selection.



Flowchart of GA

NSGA-II (Non-Dominated Sorting Genetic Algorithm) [3]

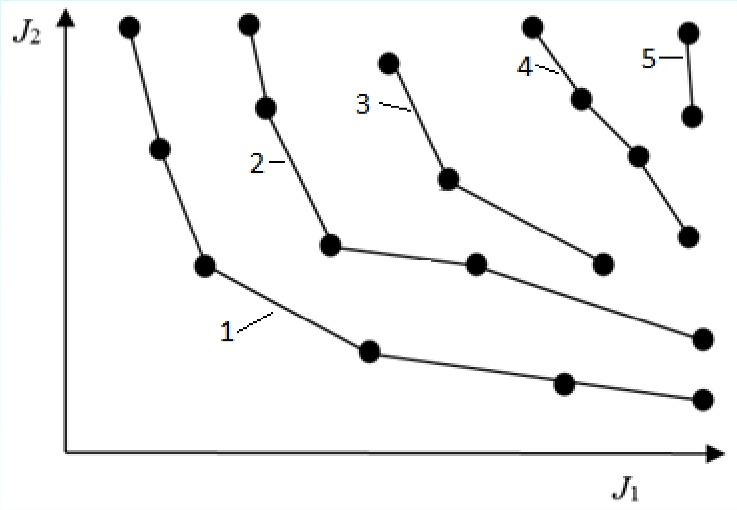
The first front = {the non-dominant set of individuals},

The second front = {the set of individuals which are dominated only by the individuals of the first front},

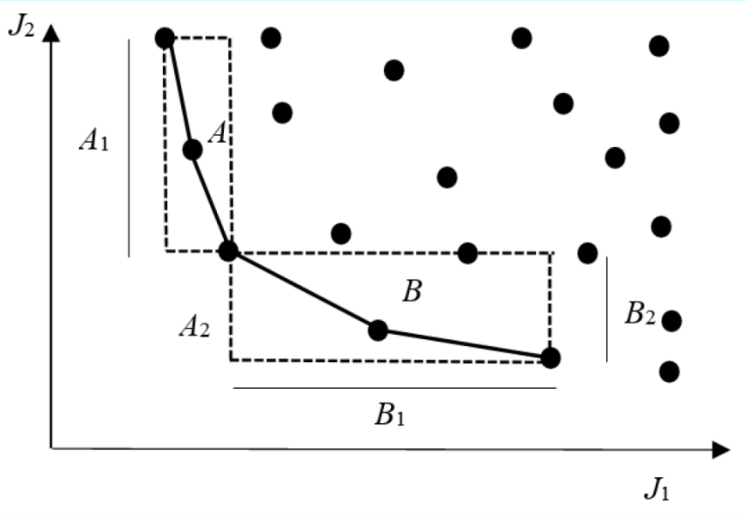
And so on.

Rank = the front number.

The **crowding distance** shows how close an individual is to its neighbors.



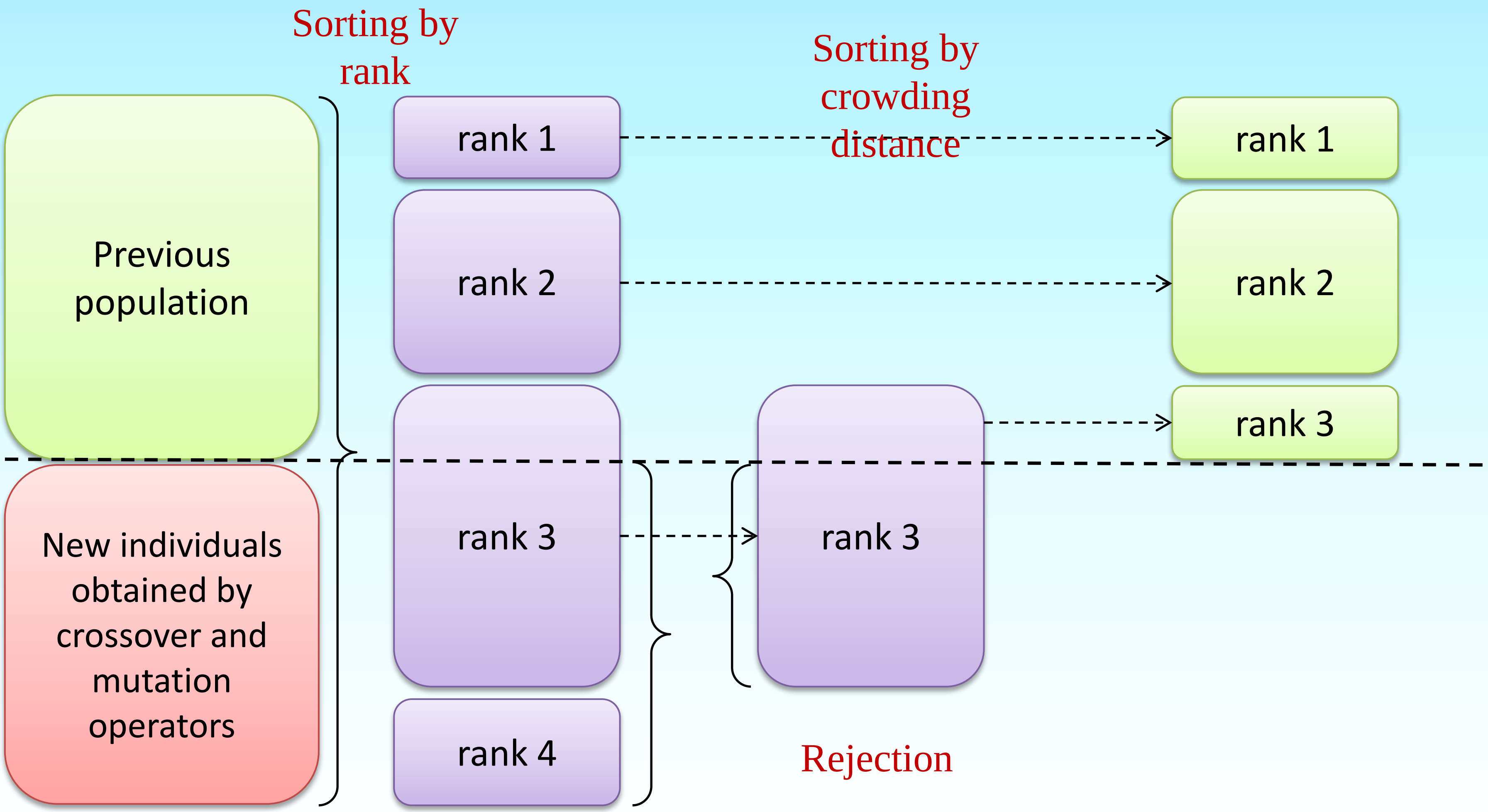
Individuals distribution by fronts



Calculation of the crowding distance

[3] Kalyanmoy Deb, Samir Agrawal, Amrit Pratap, and T Meyarivan, «A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II», Lecture Notes in Computer Science, 2000, p. 849-858.

Selection of the next generation in NSGA-II



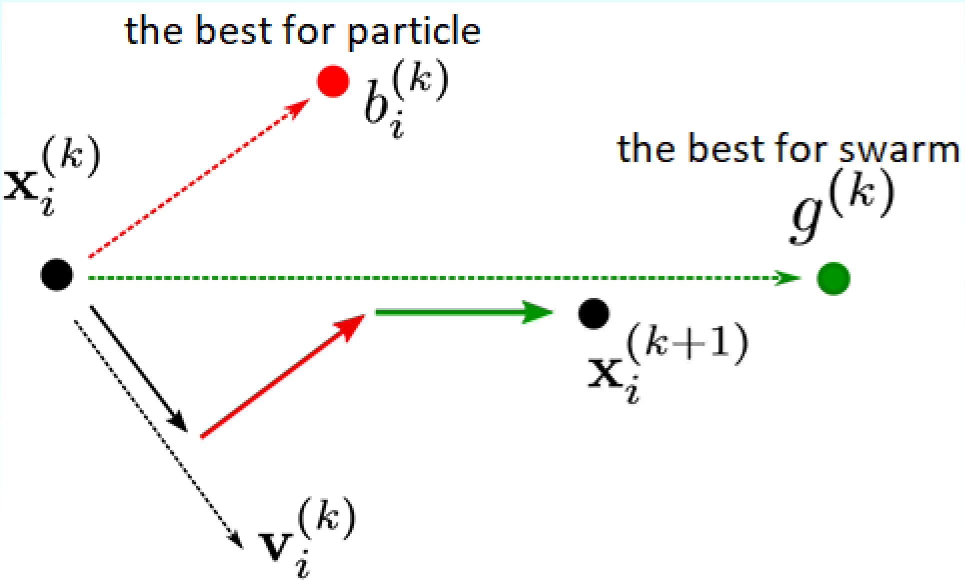
Particle swarm optimization

The **Particle swarm optimization** (PSO) is based on the simulation of the behavior of birds within a flock.

The formula for velocity of particles:

$$\boldsymbol{v}_i^{(k+1)} = C_{in}\boldsymbol{v}_i^{(k)} + C_{cog}r_1\left(\boldsymbol{b}_i^{(k)} - \boldsymbol{x}_i^{(k)}\right) + C_{soc}r_2\left(\boldsymbol{g}^{(k)} - \boldsymbol{x}_i^{(k)}\right).$$

Here k is the iteration number, $\boldsymbol{b}_i^{(k)}$ is the personal best position for the i-th particle, $\boldsymbol{g}^{(k)}$ is the best position for the whole swarm, C_{in} is the **inertia** factor, C_{cog} is the **cognitive** coefficient, C_{soc} is the **social** coefficient, r_1 and r_2 are random numbers uniformly distributed at $(0, 1)$.



Particle swarm optimization

- **Classical PSO**: C_{in}, C_{cog} and C_{soc} are random numbers uniformly distributed

$$C_{in} \in (0.1, 0.5), C_{cog} \in (1.5, 2.0) \text{ и } C_{soc} \in (1.5, 2.0).$$

- **Modified PSO** (MPSO): the particle may be affected by turbulence (the analogue of the mutation operator in GA).

Simulated annealing

$\boldsymbol{x}$ is a state of system, $f(\boldsymbol{x})$ is the system energy.
The stable crystal structure corresponds to the minimum energy $f(\boldsymbol{x})$.

1. Generation a new solution $\boldsymbol{x}'$ in accordance with the **distribution** $g(\boldsymbol{x}, T)$;
2. If $f(\boldsymbol{x}') < f(\boldsymbol{x})$, the new solution is accepted as a new state;
3. If $f(\boldsymbol{x}') > f(\boldsymbol{x})$, the new solution is accepted with the **probability**:

$$p(f, T) = \frac{1}{1 + \exp((f(\boldsymbol{x}') - f(\boldsymbol{x}))/T)}.$$

The **temperature** of the system:

$$T(k) = T_0 \alpha^k,$$

where k is the iteration step, T_0 is the initial temperature, $\alpha \in [0.5, 0.99]$ is the cooling factor.

- **Boltzmann annealing** (SA_B): $g(x'; x, T) = (2\pi T)^{-N/2} \exp(-|x' - x|^2/2T)$.
- **Cauchy annealing** (SA_C): $g(x'; x, T) = \frac{T}{(-|x' - x|^2 + T^2)^{N+1/2}}$.

N is the number of objective parameters.

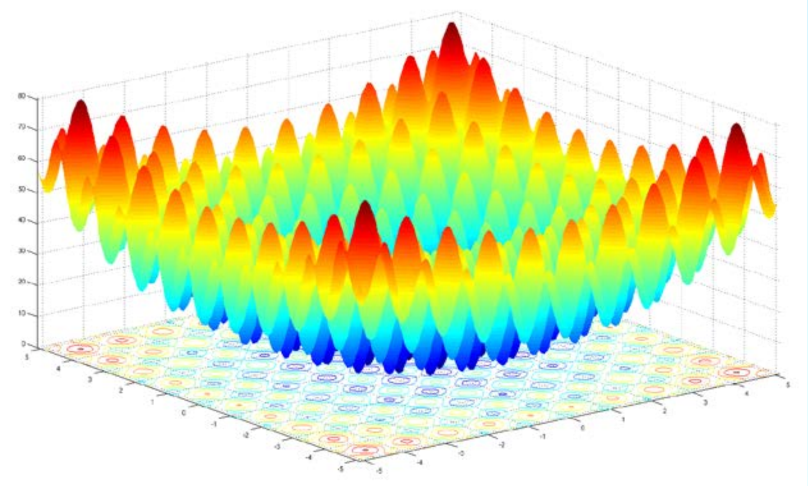
The result of the Rastrigin test optimization problem

Rastrigin's function:

$$f(\boldsymbol{x}) = 10N + \sum_{i=1}^{N}\left[x_i^2 - 10\cos(2\pi x_i)\right],$$

$$-5.12 \le x_i \le 5.12.$$

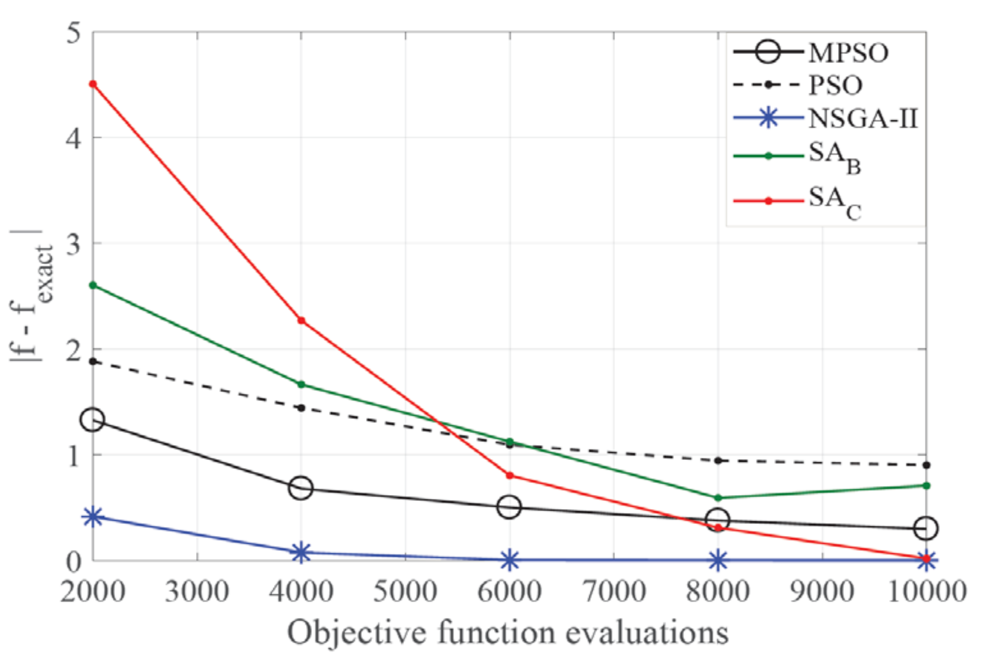
Global minimum $f(\mathbf{0}) = 0$.



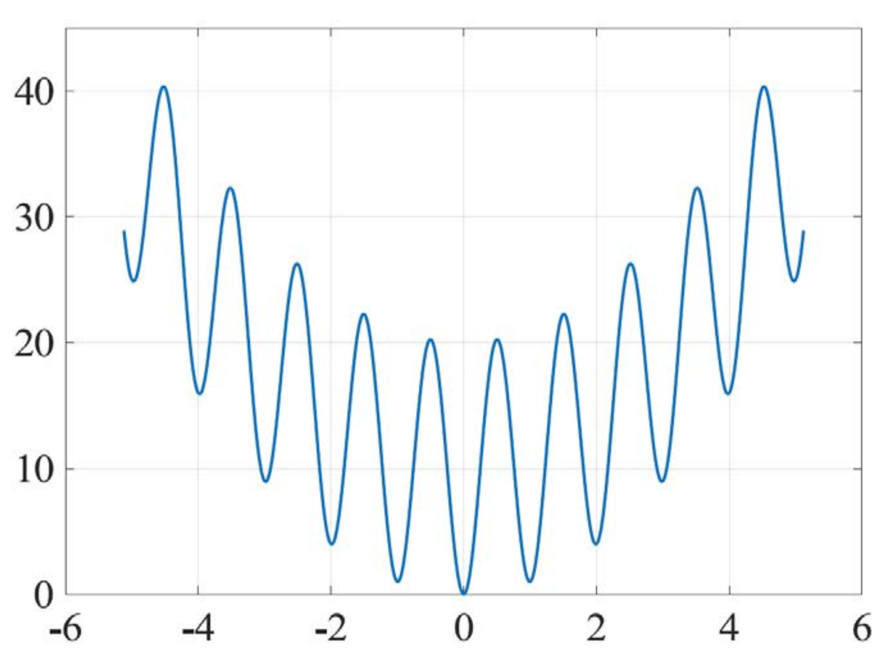
	NSGA-II	PSO	MPSO	SA_B	SA_C
$\overline{\Delta F}$	0.002246	0.795967	0.269221	0.003017	0.075076
$\bar{t}$	69.330165	0.212312	0.475257	0.273012	0.171960

Table.1. The analysis of the efficiency of the algorithms
$\overline{\Delta F}$ is an average deviation from the exact function value,
$\bar{t}$ is the average running time of the program.

Convergence of different algorithms



Convergence rate



Rastrigin's function

The result of the DTLZ4 test optimization problem

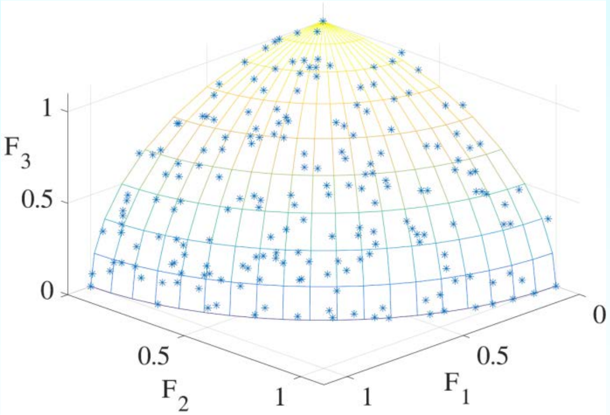
$$(F_1, F_2, F_3) \to min,$$

$$g(\boldsymbol{x}) = \sum_{i=3}^{N} (x_i - 0.5)^2, N = 12, 0 \le x_i \le 1,$$

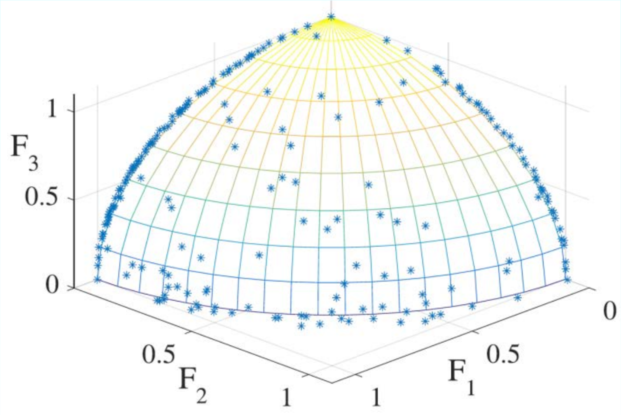
$$F_1(\boldsymbol{x}) = (1+g)\cos(0.5\pi x_1^{100})\cos(0.5\pi x_2^{100}),$$

$$F_2(\boldsymbol{x}) = (1+g)\cos(0.5\pi x_1^{100})\sin(0.5\pi x_2^{100}),$$

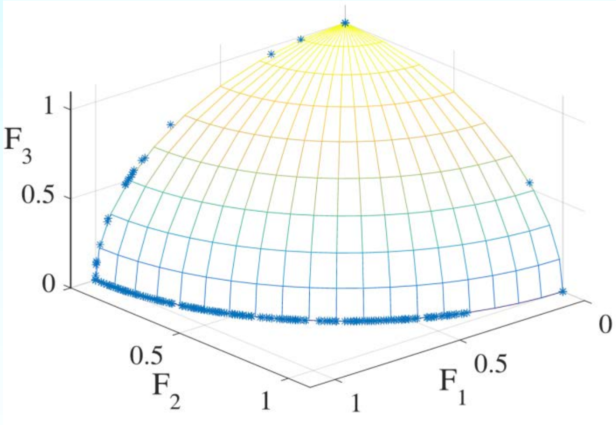
$$F_3(\boldsymbol{x}) = (1+g)\sin(0.5\pi x_1^{100}).$$



NSGA-II



PSO



SA

MFHW optimization

Task A. The single-objective optimization :

$$NPV \rightarrow max$$

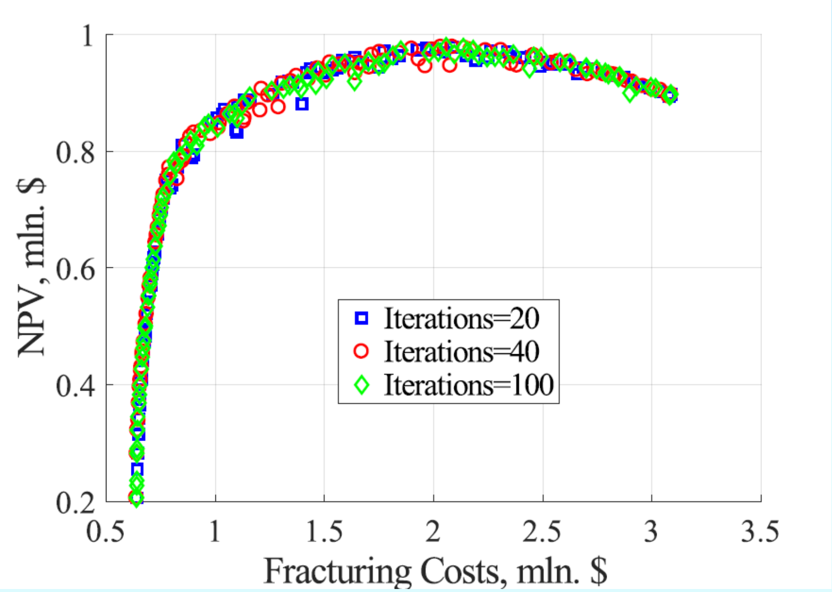
Task B. The optimization of three objective functions :

$$C_{HF} \rightarrow min, NPV \rightarrow max, Q_{tot} \rightarrow max$$

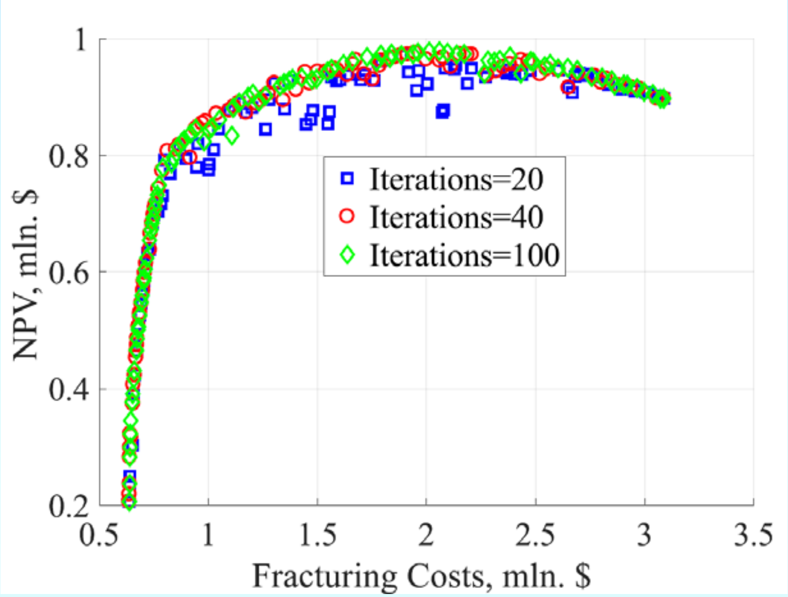
	N_f	M_p, kg	L_w, m	*NPV* , *$*	*t*
NSGA-II	9	90000	877	$9.79 \cdot 10^5$	248.66
PSO	9	90000	877	$9.79 \cdot 10^5$	239.33
MPSO	9	90000	877	$9.79 \cdot 10^5$	256.54
SA_B	9	89325	872	$9.77 \cdot 10^5$	286.15

Table.2. Results of Task A

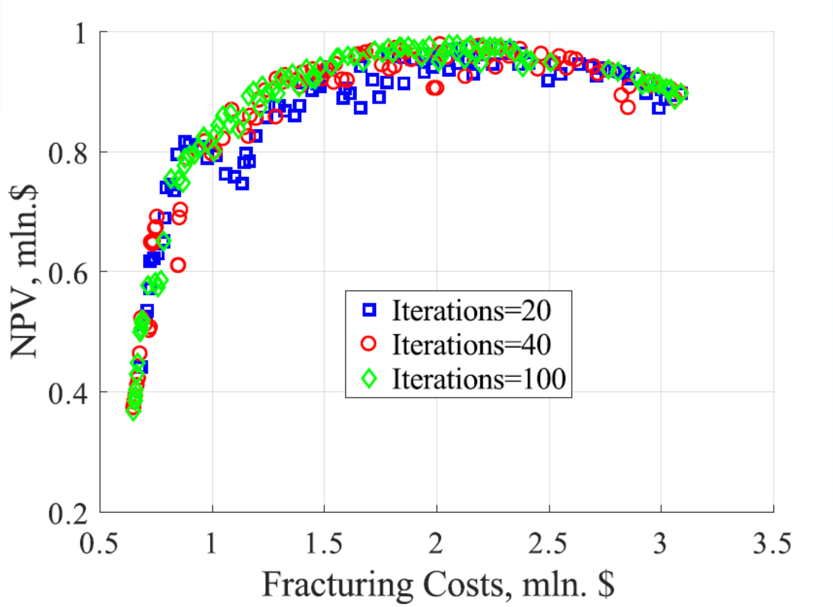
Convergence rate. Task B



NSGA-II

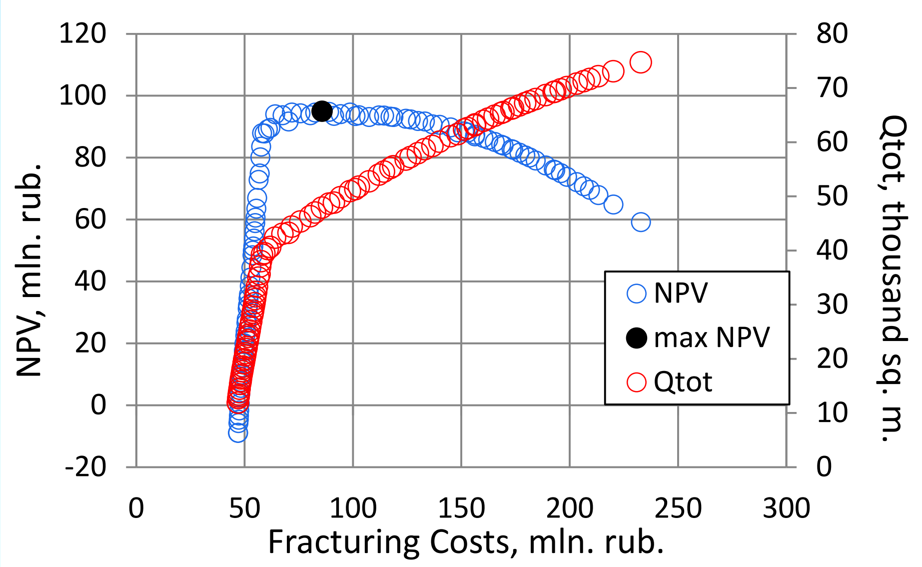


PSO

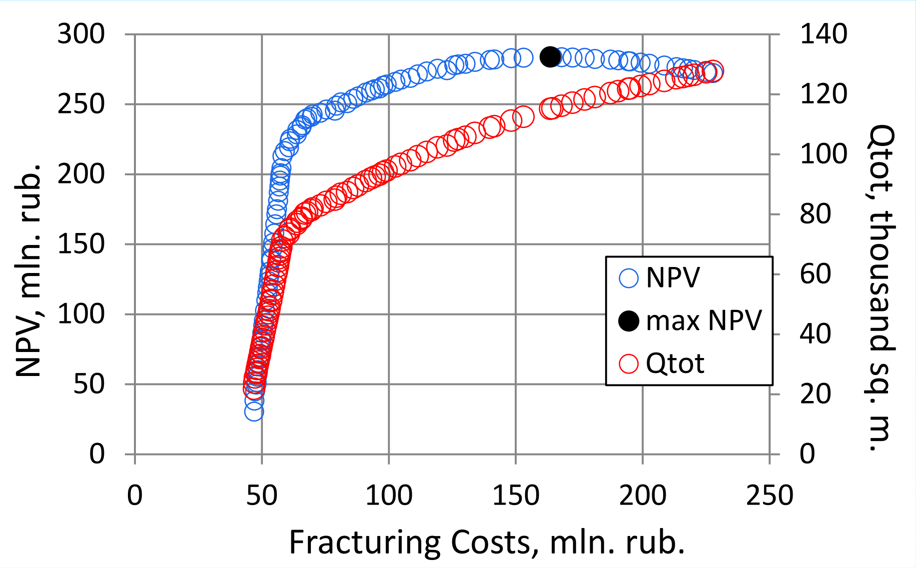


SA

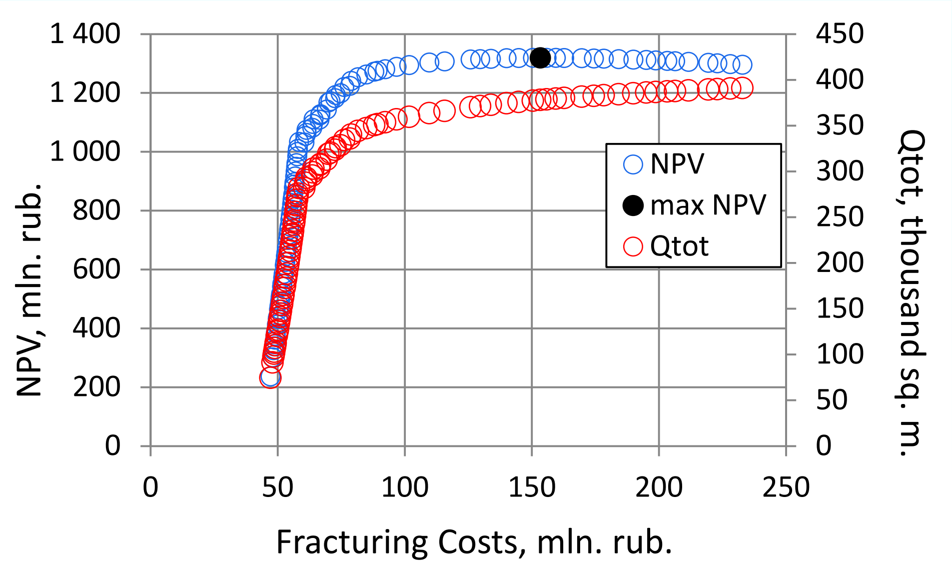
The Pareto fronts for Task B for reservoirs of different permeabilities



$K = 0.5\ mD, C = 0.2$



$K = 1\ mD, C = 0.2$



$K = 5\ mD, C = 0.4$

Conclusion

1. Considered various optimization cases:
   - the single-objective optimization,
   - the optimization of three objective functions.

2. Applied various algorithms:
   - the genetic algorithm NSGA-II,
   - the particle swarm optimization,
   - the simulated annealing.

3. The maximum NPV does not necessarily correspond to the maximum oil production. It depends on the permeability of the reservoir.

4. For MFHW optimization problem two optimization methods PSO and NSGA-II are of interest.

5. NSGA-II showed the ability to determine complex Pareto front, whereas PSO obtains the acceptable computation time.